WIND FARM LAYOUT OPTIMIZATION ON A DISCRETIZED 3D DOMAIN

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INTRODUCTION

The placement of wind turbines (WTs) in a defined region is a crucial aspect since it affects the long-term energy production of the wind farm, and thus its efficiency and economic value. In order to design an optimal layout, it is necessary to take into account topographical features, wind resource, wind turbines interactions, upfront costs, O&M and financial framework. The problem can be posed as a constrained optimization, which involves a cost function, describing the objectives, and a mathematical model describing the wake effect, the interaction between WTs, and the energy generation. Many authors have considered the design of an optimal wind farm layout [1-3]. Unfortunately, the topographical variation, which significantly affect the design of a realistic placement, are usually neglected. Moreover, most of the existing methods rely on the use of regular grids for describing the region of interest and take into account a single wind direction only.

In the following, we propose an optimization-based algorithm that, thanks to Geographic Information System (GIS) software and data, enables the design of realistic wind farm layouts over complex 3D terrains. GIS allows the description of the topography, the spatio-temporal wind speed characteristics and the presence of natural and anthropological constrains. The proposed approach relies on the solution of a sequence of Mixed Integer Linear Programs (MILP) over a discretized non-regular version of a 3D domain. In order to evaluate the proposed strategy, we consider an area in canton Neuchâtel (Switzerland) as a case study.

APPROACH

The objective is to maximise the Internal Rate of Return (IRR) of a wind farm. To do that, we look at the wind energy potential (AEP) that is defined as the total global energy content of the WTs (kWh/year). GIS software and data allow to select the areas in which the wind resource is generous and to assess the corresponding energy potential. This is achieved by combining information such as geographic position, elevation, roughness factor, etc., with climate data collected by meteorological stations. The maps of the spatial distribution of the long-term wind characteristics and, consequently the estimation of the wind energy potential are obtained using machine learning and statistical techniques [4]. All this information together allow GISs to select areas that are suitable for the installation of WTs and that satisfy potential physical, ecological, human and visual constraints. Once the candidate regions are identified, it is still necessary to capture the interaction effect between WTs, since this could strongly influence the energy content of the farm. We describe such effect using the wake model in [5]. Solving the layout optimization with this model over a continuous space would require the solution of a nonlinear optimization, which is in general very difficult. Differently from the approach in [6], where a significant approximation of the wake effect was adopted, here we rely on a linear approximation of the model in [5] and discretize the 3D domain on a grid of 50m resolution. Rather than keeping the entire regular grid, using GIS we select only the most promising cells, which correspond to the location of hilltops and ridges. Finally, to account for the spatio-temporal wind speed variability, we discretize the wind speed distribution (Weibull distribution) in bins and the wind rose in sectors. These measures allow to compute a quite realistic layout over a finite set of positions by solving a sequence of MILPs. The power curve of an ENERCON E82/3000 (80m hub height, 82m rotor diameter and 3MW rated power output) has been selected to convert the wind resource into energy production.

MAIN BODY OF ABSTRACT

In order the approach to be realistic, it is necessary to take into account all the effects that may introduce losses. The placement of wind generators can lead to aerodynamic interference between turbines. This phenomenon, called "wake effect", depends on the relative position of WTs, the wind direction, and the wind turbine model. The relationship between the speed in input to the *i-th* turbine ${V_i}^{[input]}$ and the overlap produced by the wake of the turbines in upstream is given by:

$$ {V_i}^{[input]} = {V_i}^{[inlet]} - \sqrt{\sum_{j=1}^{n} \frac{{A^j}_{overlap}}{{A^i}_{WT}} \left({V_i}^{[inlet]} - V_j(x)\right)^2} $$

where ${V_i}^{[inlet]}$ is the inlet wind speed, at the *i-th* position, in absence of interferences. $V_j(x)$ indicates the wind speed in the wake at a distance x from the *i-th* wind turbine. $\frac{{A^j}_{overlap}}{{A^i}_{WT}}$ denotes the percentage of area of the *i-th* turbine which overlaps with the wake effect of the *j-th* WT. When optimizing the turbines location, nonlinear equality constraints like the one above make the problem difficult to solve. We therefore make an approximation:

$$ {V_i}^{[input]} = {V_i}^{[inlet]} - \sum_{j=1}^{n} \sqrt{\left(\frac{{A^j}_{overlap}}{{A^i}_{WT}}\right)} \left({V_i}^{[inlet]} - V_j(x)\right) $$

We replace the non-linear term, i.e. a 2-norm of the effect of the *n* turbines in upstream, with its 1-norm. Since $\|x\|_2 \leq \|x\|_1$, the approximation leads to interference overestimation. However, thanks to the relation between norms, given *k* turbines to be installed, an optimal placement based on the 1-norm approximation leads exactly to the same result of the 2-norm. Still, the equality above remains a nonlinear function of the WTs positions due to $\frac{{A^j}_{overlap}}{{A^i}_{WT}}$. This accounts for the intersection between the set describing the wake growth of the *j-th* turbine at the location of the *i-th*, and the set describing the rotor of the *i-th*. Rather than approximating this effect, we prefer to discretize the region of interest. In this way, we are able to precompute $\frac{A^{j}{}_{overlap}}{A^{i}{}_{WT}}$ for each possible position *j* and *i* and assume them as constant in the optimization problem. With this, the placement of *k* turbines over a finite number of positions can be posed as an MILP. Binary variables are introduced for each possible position: 0/1 indicates the absence/presence of a turbine at that location. The reformulation of the constraints describing the wake effect with binaries leads to products between binary variables/binary and continuous variables. To keep the problem linear, extra variables are introduced and bilinearities replaced by linear inequality constraints [7]

$$\begin{cases} V_1{}^{[input]} = V_1^{[inlet]}\delta_1 - \sum_{j=1}^{n}\left(C_{1j}\, V_1^{[inlet]}\gamma_{1j}\right) + \sum_{j=1}^{n}\left(K_{1j}\, z_{1j}\right) \\ \vdots \\ V_M{}^{[input]} = V_1^{[inlet]}\delta_M - \sum_{j=1}^{n}\left(C_{1j}\, V_M{}^{[inlet]}\gamma_{1j}\right) + \sum_{j=1}^{n}\left(K_{Mj}\, z_{1j}\right) \end{cases}$$

$$\begin{cases} z_{ij} \le M\delta_i \\ z_{ij} \ge m\delta_i \\ z_{ij} \le V_j^{*} - m(1-\delta_i) \\ z_{ij} \ge V_j^{*} - M(1-\delta_i) \end{cases} \qquad \begin{cases} V_j{}^{[input]} \ge m\left(1-\delta_j{}^{*}\right) \\ V_j{}^{[input]} \le -\varepsilon + (M+\varepsilon)\delta_j{}^{*} \end{cases}$$

$$\begin{cases} V_j{}^{*} \le M\delta_j{}^{*} \\ V_j{}^{*} \ge m\delta_j{}^{*} \\ V_j{}^{*} \le V_j{}^{[input]} - m\left(1-\delta_j{}^{*}\right) \\ V_j{}^{*} \ge V_j{}^{[input]} - M\left(1-\delta_j{}^{*}\right) \end{cases} \qquad \begin{cases} -\delta_i + \gamma_{ij} \le 0 \\ -\delta_j + \gamma_{ij} \le 0 \\ \delta_i + \delta_j - \gamma_{ij} \le 1 \end{cases}$$

The optimal wind farm layout is obtained by maximizing $\sum_{i=1}^{M} V_i^{[input]}$ subject to the above constraints and $\sum_{i=1}^{k} \delta_i = k$, with *k* the number of turbines to be installed. Multiple wind directions and wind speed probability can be taken into account by maximizing the weighted sum of $V_i^{[input]}$ over several directions/bins. The implementation of these latter increases the number of optimization variables. Since the power curve is a monotonically increasing function of the wind speed, maximizing the sum of $V_i^{[input]}$ is equivalent to maximizing the AEP. Once the optimization is completed, it is possible to compute the IRR of the wind farm and increase the number of WTs to be installed, until the maximum IRR is attained.

Figure 1 shows in blue the “suitable” cells and in red the WTs locations, according to the MILP solution. The size of the circles corresponds to the estimated normalized AEP. The figure below is the 3D visualization of the WTs layout using 3-dimensional orthophotos.

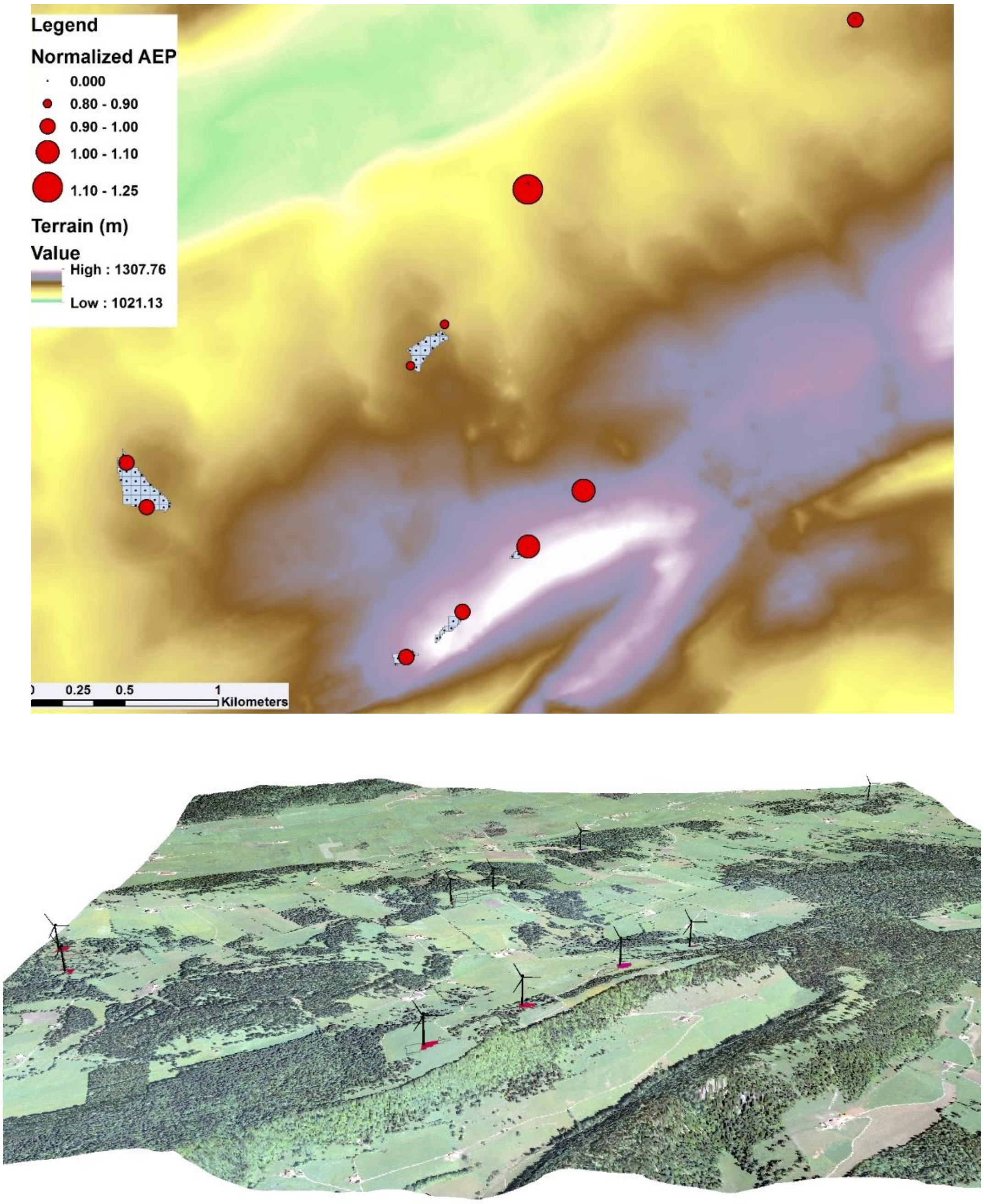

Figure 1: distribution of suitable cells (blue polygons) and optimized WTs locations with corresponding normalized AEP (above). 3D visualization of the optimized WTs positions with orthophotos (oversized WTs for better visualization).

CONCLUSION

An optimal design of a wind farm layout has been proposed. The approach is suitable for 3D terrains and allows to include important features which make the problem realistic. In particular, geodata describing the topography and the wind distribution in terms of speed and direction can be included in the optimal design. The optimal layout in terms of IRR is obtained by solving a sequence of MILPs.

LEARNING OBJECTIVE

The present work has the following objectives:

- Employment of GIS to identify suitable locations where to install wind turbines in a real environment;
- Application of mixed integer linear programming to identify the optimal location of wind turbines;
- Integration of economic model into the optimization process to estimate long-term economic performance

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