An optimal design of a wind farm layout has been proposed. The approach is suitable for 3D terrains and allows to include important features which make the problem realistic. In particular, geodata describing the topography and the wind distribution in terms of speed and direction can be included in the optimal design. The optimal layout in terms of IRR is obtained by solving a sequence of MILPs.

WIND FARM LAYOUT OPTIMIZATION ON A DISCRETIZED 3D DOMAIN

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ABSTRACT

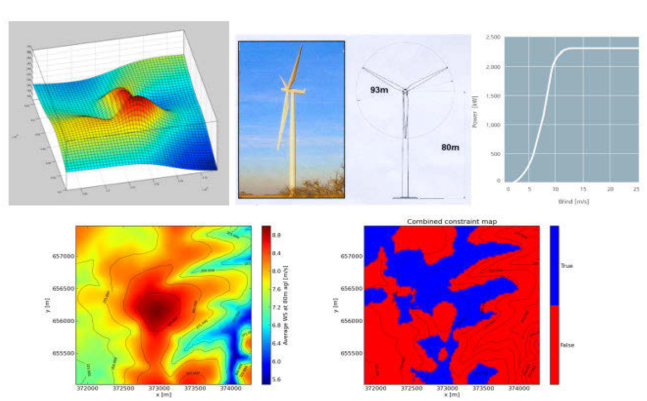
The placement of wind turbines (WTs) in a defined region is a crucial aspect since it affects the long-term energy production of the wind farm, and thus its efficiency and economic value. In order to design an optimal layout, it is necessary to take into account topographical features, wind resource, wind turbines interactions, upfront costs, O&M and financial framework. The problem can be posed as a constrained optimization, which involves a cost function, describing the objectives, and a mathematical model describing the wake effect, the interaction between WTs, and the energy generation.

APPROACH

We propose an optimization-based algorithm that, thanks to Geographic Information System (GIS) software and data, enables the design of realistic wind farm layouts over complex 3D terrains.

Elements to take into account:

- Morphology of terrain[1]
- Power Curve[2]
- Wake effect between wind turbines[3]
- Wind resource[4]
- Economic performance[5]



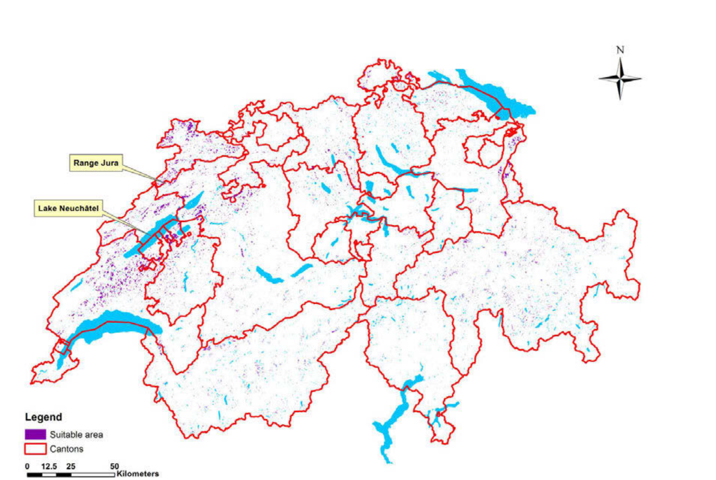


***Figure 1**: Suitable areas and morphological constraints*

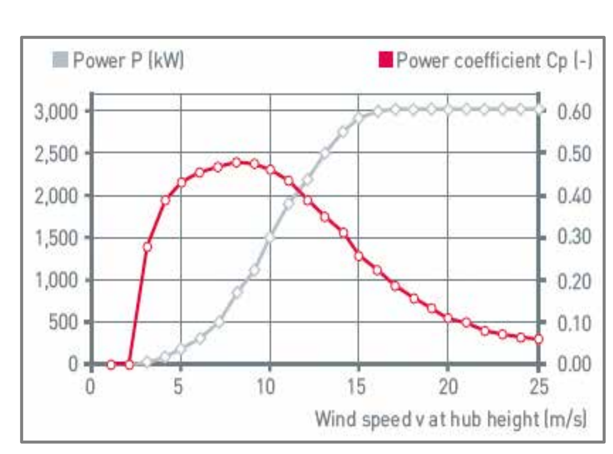


***Figure 2**:Wind turbine power curve*

$$WG_{i,wake} = D_i(\beta^{kap} + \alpha r)^{\frac{1}{kap}}$$

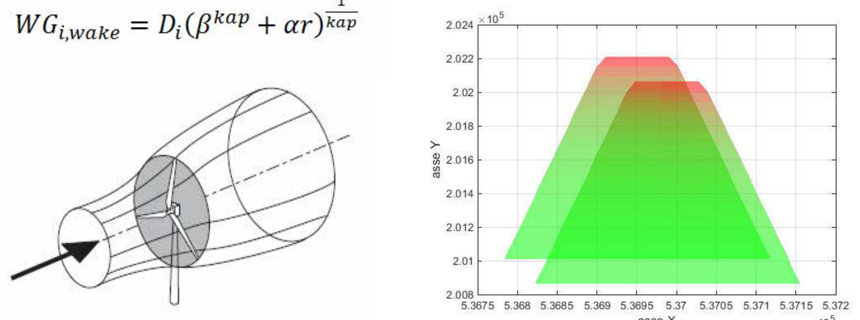
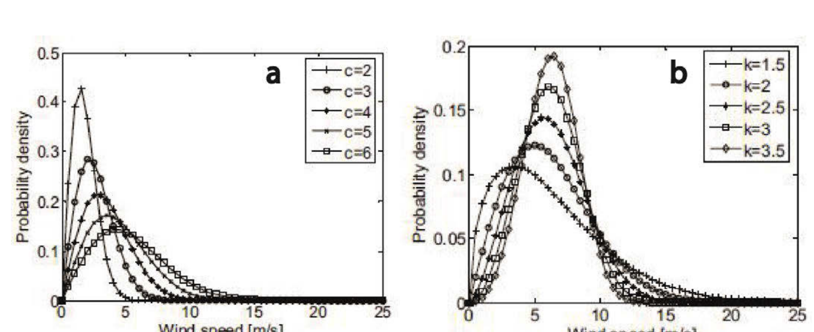


***Figure 3**: Example of "wake effect" in a wind farm offshore. Overlap of a wake effect with a WT rotor. This is a very important factor in the WT placement.*

$$AEP(WT_i) = 8760\,\beta_{avail}\,\beta_{loss}\sum_{j=1}^{N}\sum_{s=1}^{n_{dir}}\mu(\bar{u})^{(s)}P(\bar{u})^{s}$$

***Figure 5**: Annual Energy Production*



(1 e 2 from "Munteanu, I.et al. (2008). Optimal control of wind energy systems - Towards a global")

$$f(x,C,k) = \begin{cases} \frac{k}{C}\left(\frac{x}{k}\right)^{x-1}e^{(x/C)^k}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

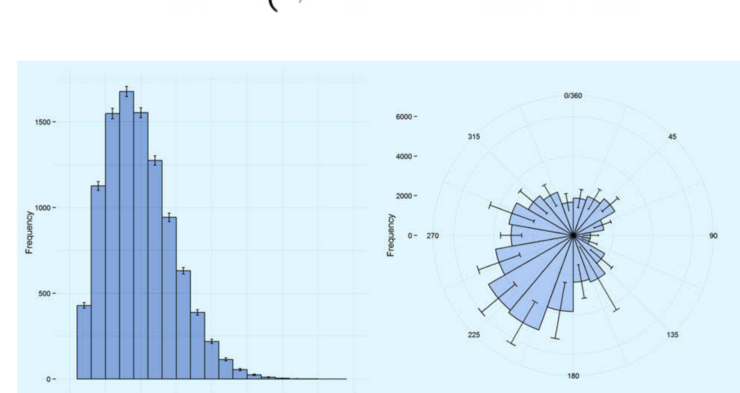


***Figure 4**: Examples of Weibull distributions (top). Wind speed and wind direction distributions discretized in bins (bottom). Note that each location comes with a different distribution.*

All this information together *allow GISs to select areas that are suitable for the installation of WTs*. Given a number of turbines to be installed, their optimal location (in terms of power production) is determined by solving a Mixed Integer Linear Programs (MILP) *over a discretized non-regular version of a 3D domain*. By iterating such approach with an increasing number of WTs to be installed, it is possible to obtain the placement which maximises the return of investment of the farm.

ALGORITHM

The relationship that describes the aerodynamic interference between the speed in input to the *i-th* turbine and the overlap produced by the wake of the turbines in upstream is given by:

$$V_i^{[input]} = V_i^{[inlet]} - \sqrt{\sum_{j=1}^{n}\left(\frac{A^j{}_{overlap}}{A^i{}_{WT}}\right)\left(V_i^{[inlet]} - V_j(x)\right)^2}$$

Nonlinear equality constraints like the one above make the problem difficult to solve. We therefore make an approximation:

$$V_i^{[input]} = V_i^{[inlet]} - \sum_{j=1}^{n}\sqrt{\left(\frac{A^j{}_{overlap}}{A^i{}_{WT}}\right)}\left(V_i^{[inlet]} - V_j(x)\right)$$

Since $\|(x+y)\|_2 \leq \|(x+y)\|_1$ he use of the 1-norm instead of the 2-norm does not have any repercussion on the optimal placement. Binary variables are then introduced to indicate the presence/absence of a wind turbine at a given location

$$\begin{cases} V_1^{[input]} = V_1^{[inlet]}\delta_1 - \sum_{j=1}^{n}(C_{1j}V_1^{[inlet]}\gamma_{1j}) + \sum_{j=1}^{n}(K_{1j}z_{1j}) \\ \vdots \\ V_N^{[input]} = V_1^{[inlet]}\delta_N - \sum_{j=1}^{n}(C_{1j}V_N^{[inlet]}\gamma_{1j}) + \sum_{j=1}^{n}(K_{Nj}z_{1j}) \end{cases}$$

The reformulation comes with a set of linear constraints which involve both continuous and binary variables

$$V_j^* = max(V_j^{[input]}, 0)$$

$$V_j^* = V_j^{[input]}\delta_i^*$$

$$\begin{cases} V_j^* \leq M\delta_j^* \\ V_j^* \geq m\delta_j^* \\ V_j^* \leq V_j^{[input]} - m(1-\delta_j^*) \\ V_j^* \geq V_j^{[input]} - M(1-\delta_j^*) \end{cases}$$

$$z_{ij} = V_j^*\delta_i$$

$$\begin{cases} z_{ij} \leq M\delta_i \\ z_{ij} \geq m\delta_i \\ z_{ij} \leq V_j^* - m(1-\delta_i) \\ z_{ij} \geq V_j^* - M(1-\delta_i) \end{cases}$$

$$\gamma_{ij} = \delta_i\delta_j$$

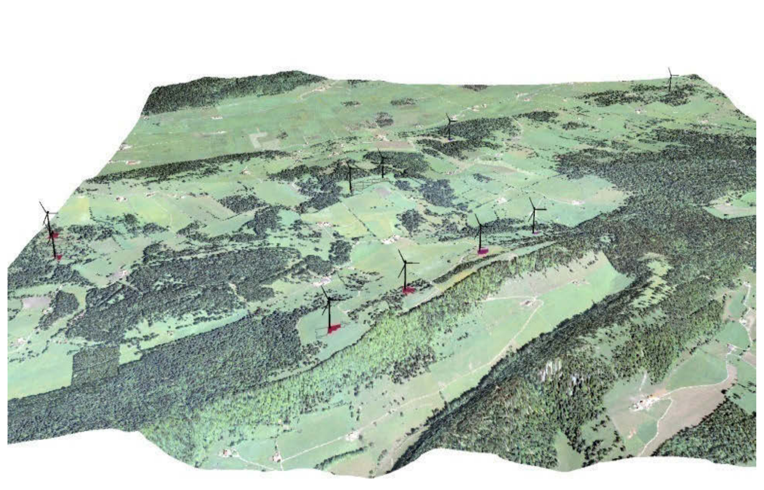
$$\begin{cases} -\delta_i + \gamma_{ij} \leq 0 \\ -\delta_j + \gamma_{ij} \leq 0 \\ \delta_i + \delta_j - \gamma_{ij} \leq 1 \end{cases}$$

$$\begin{cases} V_j^{[input]} \geq m(1-\delta_j^*) \\ V_j^{[input]} \leq -\varepsilon + (M+\varepsilon)\delta_j^* \end{cases}$$

This reformulation allows to pose the problem of maximizing the energy production as an MILP with cost function $\sum_{i=1}^{M} V_i^{[input]}$. The evaluation of the IRR is done a posteriori. By iterating the approach with an increasing number of WT, it is possible to obtain the optimal location and WTs to be installed.

CONCLUSIONS

The following figures show the optimal placement in terms of IRR using the approach described above on a real wind farm.



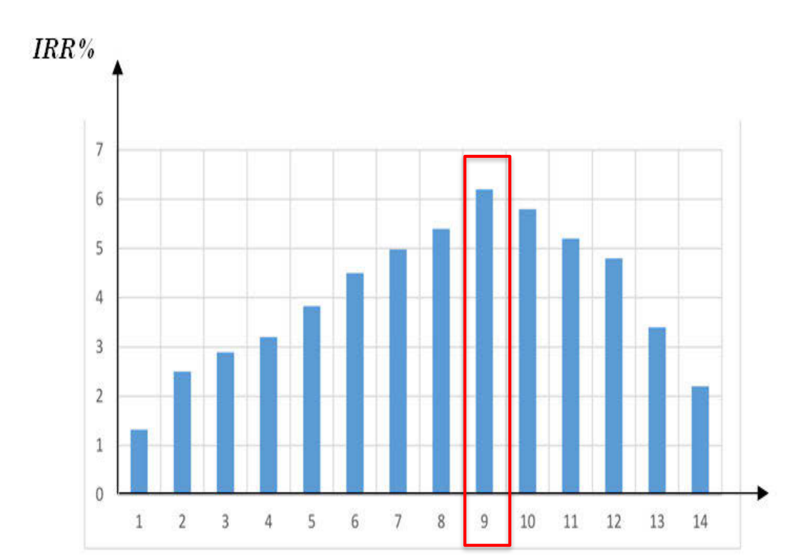


Figure 1: Visualization of the optimized WTs positions with orthophotos (oversized WTs for better visualization). Economic performance:IRR trend as a function of number of wind turbines installed.

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