DEFINING THE WAKE DECAY CONSTANT AS A FUNCTION OF TURBULENCE INTENSITY TO MODEL WAKE LOSSES IN ONSHORE WIND FARMS

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Abstract

Modelling the wake effect generated by wind turbines is an essential part for calculating a wind farm’s expected energy production. Operating wind turbines disturb the flow of the wind, which results in decreased production of the downwind turbines. The N. O. Jensen model is an industry standard wake model, which has only one adjustable parameter – the wake decay constant ($k_w$). This parameter defines the expansion rate of the generated wake, and has traditionally been derived semi-empirically based on the surface roughness. A clear link between $k_w$ and the ambient turbulence intensity ($TI$) is though expected: high ambient turbulence leads to a faster decay of the generated wake, and therefore to lower wake losses, and vice-versa. Since the influence of the roughness on the ambient turbulence intensity is expected to be less significant at higher heights, the Jensen model using $TI$-based $k_w$ values should show a better performance at higher hub heights than using the traditional roughness-based $k_w$ values. This hypothesis is investigated in this study by comparing observed and modelled wake losses based on different $k_w$ values. Two case studies are analysed based on operational data from an onshore wind farm. The results show that the goodness of fit between modelled and observed wake losses has a clear dependency on the wind speed. At higher wind speeds, the $TI$-based wake decay constant resulted in a better accuracy of the modelled wake loss as compared to the roughness-based wake decay constant, while for lower wind speeds the N. O. Jensen model performed most accurately when using the roughness-based wake decay constant of 0.075 typically used for onshore wind farms.

Keywords: N. O. Jensen model, Wake loss, Wake Decay Constant, Turbulence intensity, Onshore wind farms, Forested terrain

1. Introduction

The N.O. Jensen wake model is an industry standard model for simulating wake-induced wind speed deficits. Production losses caused by wake effects are typically of the order of 5% to 20% of the annual energy production [2]. Embedded in the windPRO software, the N. O. Jensen model has only one adjustable parameter – the wake decay coefficient ($k_w$). The standard settings presented in windPRO define fixed $k_w$ values for different types of landscapes and are derived from terrain roughness classifications. However, since the wake decay constant defines the expansion rate of the wake, it is strongly linked to the ambient turbulence intensity ($TI$): high ambient turbulence leads to a faster decay of the generated wake and therefore to lower wake losses and vice-versa. Since the roughness-based values were defined, the average hub height of onshore turbines has been increasing considerably, implying a weaker dependence of the wake loss on the surface roughness. Therefore, it is hypothesised that using the standard roughness-based $k_w$ values instead of $TI$-based $k_w$ will cause the N. O. Jensen model to underestimate the real wake losses when applying the model to higher hub heights.
The present study investigates how the relation of wake decay and turbulence intensity affects the predictive capability of the Jensen model. Two case studies are investigated based on operational data from an onshore wind farm located in Sweden operated by Stena Renewable: Case study I corresponds to a set of two turbines with a hub height of 80 m; Case study II corresponds to a set of two turbines with 105 m hub height. By comparing the observed wake with the wake modelled based on different $k_w$ values, it is investigated whether a TI-based $k_w$ results in a better goodness of fit between modelled and observed wake loss.

2. Background

The amount of research investigating the definition of the $k_w$ as function of TI is very limited and has mostly concerned offshore sites [1, 3]. The main focus of the existing publications has been the adjustment of the $k_w$ until it matched observed power losses, as well as the combination of the obtained results into model definitions. A first onshore case study was conducted by [4], who investigated the Sexbierum wind farm that is located in flat and homogenous terrain. They conclude that the Jensen model is able to outperform even the more comprehensive wake models in terms of accuracy when a TI-based $k_w$ is employed [4]. The relation $k_w = k \cdot TI$, where $k$ is the dimensionless von Kármán constant ($k = 0.41$) was found valid for hub heights of 40 m - 60 m during stable conditions, and at heights above 100 m in neutral and unstable conditions [4]. The windPRO recommendation of $k_w = 0.5 \cdot TI$ is valid under neutral atmospheric conditions and has been used in other studies [3]. For further information on how this relation is derived the reader is referred to [4].

In the following, the validity of $k_w \approx 0.5 \cdot TI$ for different hub heights (80 m and 105 m) and wind speed intervals is tested. The analysed wind farm is located in semi-complex and forested terrain.

2.1 Data

Three years of data from the wind farm’s Supervisory Control and Data Acquisition (SCADA) were made available by the wind farm owner and operator Stena Renewable AB. The data was provided for eleven variables as 10-min mean values and covers the period from 01.01.2013 to 26.03.2016. The full years 2013, 2014 and 2015 were included in the subsequent analysis. As a next step the data was filtered for erroneous values and non-full performance. An example for such events are values of the wind direction not between 0° and 360° degrees, or power production data that is significantly higher than the rated power. To avoid disturbances such as icing, a temperature filter was applied to include only data measured at $> 7°$ C, which is considered a reasonable temperature limit. Another important step was the time stamp intersection of the turbine data used for the subsequent case studies. This ensures that only the same points in time are investigated for both turbines. Moreover, the obtained time stamps were then intersected with the modelled production time series, again to ensure that the measured and modelled data sets have the same time stamps.

2.2 Wake Decay Constant Selection

The Jensen model in the PARK module allows the input of roughness-based $k_w$ values as well as a TI-based $k_w = 0.5 \cdot TI$. For the following study, three Jensen wake loss simulations were conducted with different $k_w$. The first simulation used $k_w = 0.075$, which is the value typically used for onshore wind farm wake loss simulations. For the second simulation $k_w = 0.1$ was used. It was derived from the site’s specific roughness. A final simulation was then conducted using $k_w = 0.5 \cdot TI$, where TI was calculated from the available wind measurements.

3. Results

To compare the modelled wake losses with the observations, the ratio between the power of the downwind turbine T2 and the upwind turbine T1 for the wind direction interval in which the wake occurs is plotted. It was decided to define the wake case as occurring most clearly when the production (P) of the upwind turbine T1 ranges between 300 kW - 1900 kW. To ensure a good
wake resolution, the relative power is grouped in 5° wind direction bins and the mean of each bin is plotted as error bar. Furthermore, to determine if the results depend on the prevailing wind speeds, different power intervals are investigated. The chosen power thresholds and their respective wind speeds are shown in Table 1.

Table 1: Wind Speed and Corresponding Power Output

<table>
<thead>
<tr>
<th>Power [kW]</th>
<th>Wind Speed [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>5.8</td>
</tr>
<tr>
<td>500</td>
<td>6.5</td>
</tr>
<tr>
<td>700</td>
<td>7.8</td>
</tr>
<tr>
<td>900</td>
<td>8.8</td>
</tr>
<tr>
<td>1900</td>
<td>8.9 - 11</td>
</tr>
</tbody>
</table>

3.1 Case I

In Figure 1, the observed and the modelled wake loss are plotted. Whereas the left side of the curves indicate a very good fit between the modelled and observed wake, a larger difference is seen for the central and right-hand parts. The individual curves start to divert at around 280°. At 290° the observed maximum power deficit suggests that T2 produces only around 36% of the power T1 produces. Using the industry standard $k_w = 0.075$ suggests a slightly lower deficit of about 35%. The simulation with the TI-based $k_w$ is shown in red. It shows a larger discrepancy, which suggests that T2 only produces 30% of the power T1 produces. The largest discrepancy is obtained when simulating the wake with a purely roughness dependent $k_w = 0.1$. At 295° the curves of the modelled power deficit start to align with each other, however showing a difference to the observed wake. The asymmetry between modelled and observed wake is likely to be the result of terrain irregularities that causes a divergence of the flow.

![WDC Comparison at 300 kW < P1 < 1900 kW](image)

Figure 1: Relative Power Deficit between Observed and Modelled Wake Loss at 80 m Hub Height
• Using \( k_w = 0.075 \) shows the best fit with the observed power.
• The goodness of fit is quantified by using the mean average error (MAE). The MAE values in Table 2 show the goodness of fit: the lowest value represents the best fit and the highest value the largest error between model and observation.
• Comparing the numbers it is evident that the observed power deficit is best modelled using \( k_w = 0.075 \). Using \( k_w = 0.5 \) TI results in a slightly lower goodness of fit and using \( k_w = 0.1 \) gives the worst fit of the compared cases.

<p>| Table 2: Mean Absolute Error Figure 1 |</p>
<table>
<thead>
<tr>
<th>( k_w )</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075</td>
<td>0.036</td>
</tr>
<tr>
<td>0.1</td>
<td>0.047</td>
</tr>
<tr>
<td>0.5 TI</td>
<td>0.039</td>
</tr>
</tbody>
</table>

<p>| Table 3: Mean Absolute Error Figure 2 |</p>
<table>
<thead>
<tr>
<th>( k_w )</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075</td>
<td>0.056</td>
<td>0.053</td>
<td>0.047</td>
<td>0.04</td>
</tr>
<tr>
<td>0.1</td>
<td>0.056</td>
<td>0.052</td>
<td>0.051</td>
<td>0.053</td>
</tr>
<tr>
<td>0.5 TI</td>
<td>0.062</td>
<td>0.054</td>
<td>0.051</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Table 2 shows the mean absolute error for different values of \( k_w \). Table 3 shows the mean absolute error for different power intervals (a, b, c, d) at 80 m Hub Height.

Figure 2 (a) – (d) and Table 3 show the results considering specific power bins.

Figure 2: Relative Power Deficit between Observed and Modelled Wake Loss for Different Power Production Intervals at 80 m Hub Height.
• At 300 kW - 500 kW, the best agreement between modelled and measured wake is obtained for $k_w = 0.075$, however it is immediately followed by the roughness-based $k_w = 0.1$.
• When $P_1$ is between 500 kW - 700 kW, and 700 kW - 900 kW, the best agreement is obtained for $k_w = 0.075$. The deviation obtained using $k_w = 0.5 \cdot TI$ is just slightly higher. Setting $k_w = 0.1$ still gives the largest deviation between modelled and measured wake loss.
• For the interval 900 kW < $P_1$ < 1900 kW, the best fit is achieved for $k_w = 0.5 \cdot TI$.

3.2 Case II
The second case study investigates a wake case occurring at 105 m hub height. The turbine spacing is 5 D. From a first look at Figure 3 it can be observed that the overall maximum power deficit is ≈ 40%, that there is a wake asymmetry (i.e. better fit with observations in the right side of the curves), and that at 265° T2 produces more power than T1.

![WDC Comparison at 300 kW < P1 < 1900 kW](image)

**Figure 3**: Relative Power Deficit between Observed and Modelled Wake Loss at 105 m Hub Height

• There is a minor difference between using $k_w = 0.5 \cdot TI$ and $k_w = 0.075$
• $k_w = 0.1$ shows the largest discrepancy
• From Table 4 it is clear that the lowest MAE is obtained using $k_w = 0.075$. When comparing the MAE range from the first case with the values from Table 4, it can be seen that the range between those values is smaller, which may suggest that the accuracy of the Jensen model is less dependent on the used $k_w$ in the second case study.

<table>
<thead>
<tr>
<th>$k_w$</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075</td>
<td>0.046</td>
</tr>
<tr>
<td>0.1</td>
<td>0.052</td>
</tr>
<tr>
<td>0.5 TI</td>
<td>0.048</td>
</tr>
</tbody>
</table>

**Table 4**: Mean Absolute Error Figure 3

<table>
<thead>
<tr>
<th>$k_w$</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075</td>
<td>0.063</td>
<td>0.065</td>
<td>0.059</td>
<td>0.036</td>
</tr>
<tr>
<td>0.1</td>
<td>0.067</td>
<td>0.070</td>
<td>0.065</td>
<td>0.039</td>
</tr>
<tr>
<td>0.5 TI</td>
<td>0.068</td>
<td>0.070</td>
<td>0.057</td>
<td>0.035</td>
</tr>
</tbody>
</table>

**Table 5**: Mean Absolute Error Figure 4

Again, different power production / wind speed bins are considered. The results are shown in Figure 4 (a) – (d) and Table 5.
At 300 kW < P1 < 500 kW, the difference between \( k_w = 0.1 \) and \( k_w = 0.5 \cdot \text{TI} \) is almost negligible when considering the MAE for the whole wake.

The standard value \( k_w = 0.075 \) gives the lowest error. However, when considering the maximum power deficit at 280°, the standard value and the TI-based value show only a slight discrepancy.

The same observations can be made when the power limits are set to 500 kW < P1 < 700 kW.

At the next interval, 700 kW < P1 < 900 kW, \( k_w = 0.075 \) and \( k_w = 0.5 \cdot \text{TI} \) are closer together.

As opposed to the results of the first case study, the modelled losses do not deviate as strongly from the observations when plotted for the interval 900 kW < P1 < 1900 kW. However, the overall MAE is lower and does not change considerably among the different \( k_w \) values. Similar to the first case study, \( k_w = 0.5 \cdot \text{TI} \) does perform at a lower MAE compared to the other values for this bin, however the difference is marginal.

4. Dependency on Turbulence Intensity

To determine whether the accuracy of the model depends on the TI, the percentage change was plotted. The TI was binned in 1σ intervals at ± 3 σ around its mean. Figure 5 shows the
percentage change between modelled and observed wake loss normalised to the observed losses. It can be seen that there is no clear relation between the model accuracy and the TI when considering the whole TI distribution.

![Difference of TI bins at 300kW < P1 < 1900kW](image)

**Figure 5:** Difference between Modelled and Observed Relative Power, Normalised to the Observed Losses

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### 5. Sensitivity and Limitations

The presented results are based on wind speed measurements that were taken with nacelle anemometers and therefore bare an uncertainty. Therefore a sensitivity study was conducted. A virtual calibration function was derived to adjust the nacelle anemometer measurements for scale and offset and rerun the wake loss simulation of the first case. This virtual function was derived from properties of available calibration functions from two adjacent turbines, which were obtained with nacelle mounted LiDAR systems. A linear fit with a scale of 1.15 and offset of 0.5 was used on the wind speed measurements of T1. As a result, Figure 6 was produced. The dashed lines represent the simulations with the adjusted wind speeds.

![WDC Comparison at 300 kW < P1 < 1900 kW](image)

**Figure 6:** Comparison of Relative Power Deficit between Observed and Modelled Wake Loss at 80 m Hub Height
The input data of the non-adjusted simulations are the same as those made from analysing Figure 1. However, when considering the wind speed adjusted values it can be seen that the simulation using $k_w = 0.5$ TI results in a significant under prediction. The relative power in the observed wake is around 0.38, which corresponds to an overall power deficit at T2 of 62%. However, the simulation suggests that the wake loss is approximately 55%. Using $k_w = 0.075$ does not show such a large deviation between results of the adjusted wind speed and the measured wind speed. Consequently, when modelling wake losses with a turbulence-based $k_w$ the results are much more volatile to a change in wind speed. For the present study this is rather obvious as the TI is calculated from the measured wind speed.

6. Conclusions

In the presented case studies, the N.O. Jensen model performed most accurately when using $k_w = 0.075$ compared to using a TI-based value when considering all power production / wind speed intervals. However, the goodness of fit between the different modelled and observed losses showed a dependency on the power production / wind speed interval. At higher wind speeds, the TI-based $k_w$ showed the lowest error of the compared cases, while for lower wind speeds the N. O. Jensen model performed most accurately when using $k_w = 0.075$. The main source of uncertainty in the results of this study is related to the uncertainty in the used wind speed measurements, which has a larger impact on the wake model when using a TI-based $k_w$ than using a roughness-based $k_w$. Moreover, it can be concluded that the goodness of the fit between modelled and observed wake loss does not depend on the TI. These conclusions are valid for single wake cases of turbines located in semi-complex and forested terrain and should be verified and extended by further research to allow generalisations.

This paper is an excerpt from Joachanan Kollwitz’s Master thesis, which will be published through the DiVA portal under the same title. For questions please contact Mr. Kollwitz at: jochanan.kollwitz@gmail.com

References